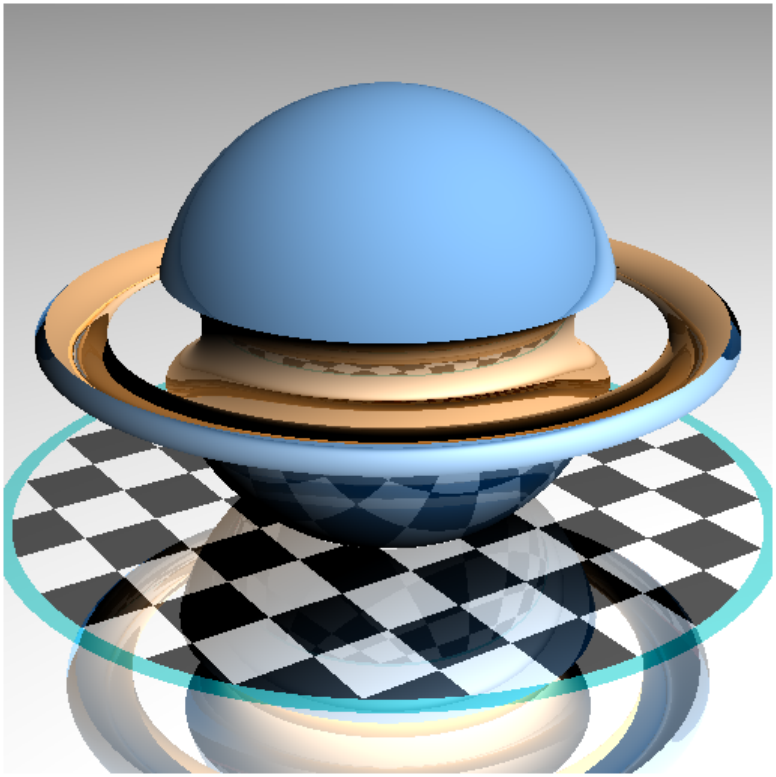
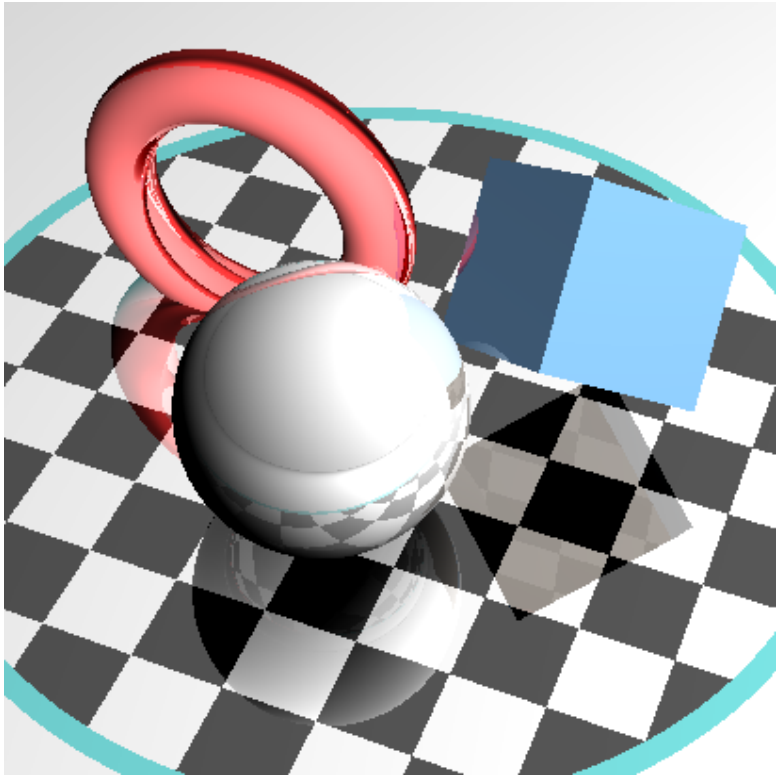
Advanced Graphics





Ray Tracing: Geometry and Lighting

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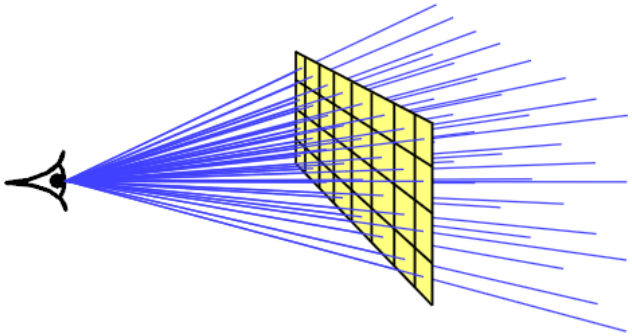
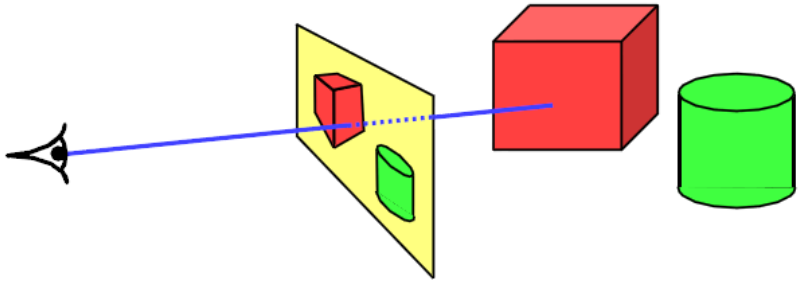
Ray tracing revisited

Ray tracing

- a powerful alternative to polygon scan-conversion techniques
- given a set of 3D objects, shoot a ray from the eye through the centre of every pixel and see what it hits



shoot a ray through each pixel



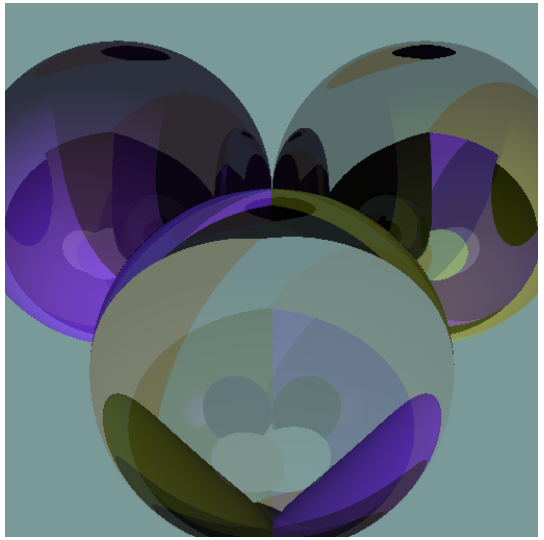
whatever the ray hits determines the colour of that pixel

(Slide from Neil Dodgson's *Computer Graphics and Image Processing* notes, Cambridge University.)

Ray tracing

The basic algorithm is straightforward, but there's much room for subtlety

- Refraction
- Reflection
- Shadows
- Anti-aliasing
- Blurred edges, depth-of-field effects



```
typedef struct{double x,y,z;}vec;vec U,black,amb={.02,.02,.
02};
struct sphere{vec cen,color;double rad,kd,ks,kt,kl,ir;}*s,
*best
,sph[]={0.,6.,.5,1.,1.,1.,.9,.05,.2,.85,0.,1.7,-1.,8.,-.
5,1.,.5
,.2,1.,.7,.3,0.,.05,1.2,1.,8.,-.5,.1,.8,.8,1.,.3,.7,0.,0.,
1.2,3
.,-6.,15.,1.,.8,1.,7.,0.,0.,0.,.6,1.5,-3.,-3.,12.,.8,1.,1.,
5.,0
.,0.,0.,.5,1.5,};int yx;double u,b,tmin,sqrt(),tan();double
vdot(vec A,vec B){return A.x*B.x+A.y*B.y+A.z*B.z;}vec vcomb(
double a,vec A,vec B){B.x+=a*A.x;B.y+=a*A.y;B.z+=a*A.z;return
B;}vec vunit(vec A){return vcomb(1./sqrt(vdot(A,A)),A,
black);}
struct sphere*intersect(vec P,vec D){best=0;tmin=10000;
s=sph+5;
while(s-->sph)b=vdot(D,U=vcomb(-1.,P,s->cen)),u=b*b-vdot(U,U)
+
s->rad*s->rad,u=u>0?sqrt(u):10000,u=b-u>0.000001?b-u:b+u,
tmin=
u>0.00001&&u<tmin?best=s,u:tmin;return best;}vec trace(int
level,vec P,vec D){double d,eta,e;vec N,color;struct
sphere*s,
*l;if(!level--)return black;if(s=intersect(P,D));else return
amb;color=amb;eta=s->ir;d=-vdot(D,N=vunit(vcomb(-1.,P=vcomb(
tmin,D,P),s->cen)));if(d<0)N=vcomb(-1.,N,black),eta=1/eta,d=
-d;l=sph+5;while(l-->sph)if((e=l->kl*vdot(N,U=vunit(vcomb
(-1.,P
,l->cen))))>0&&intersect(P,U)==l)color=vcomb(e,l->color,
```

Paul Heckbert's 'minray' ray tracer, which fit on the back of his business card. (circa 1983)

Ray tracing

The ray tracing time for a scene is a function of
(num rays cast) x
(num lights) x
(num objects in scene) x
(num reflective surfaces) x
(ray reflection depth) x …

Image by nVidia

Contrast this to polygon rasterization: time is a function of the number of elements in the scene times the number of lights.

The algorithm

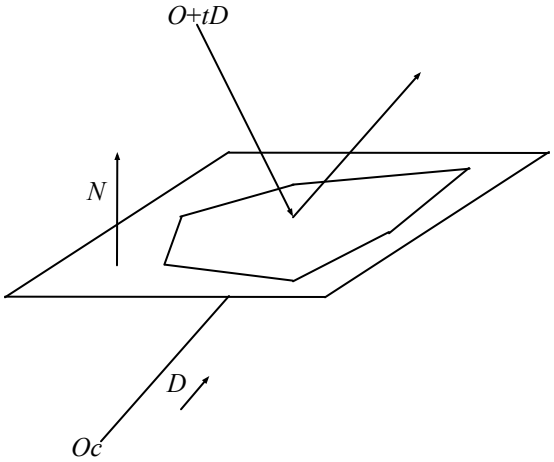
For each pixel on the screen, do:

a. Calculate ray from eye (O) through pixel (X)
   i. Set D = (X-O) / |(X-O)|
   ii. Ray: $R=O+tD$
b. Find ray/primitive hit point (P) and normal (N)
c. Compute shadow, reflection, transparency rays; recursively call steps *a,b,c*
d. Calculate lighting of surface at point

Ray/plane intersection

Ray $R=O+tD$

Poly $P=\{v^1,\ldots,v^n\}$



$N= (v^n-v^1)\times(v^2-v^1)$

$N\bullet(O+tD-v^1)=0$

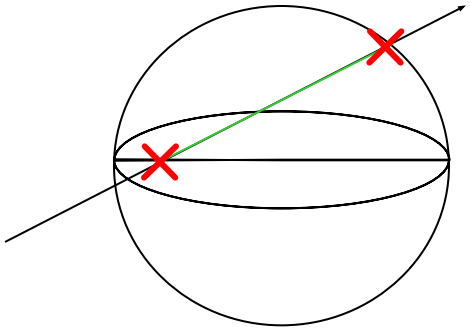
$N_x(O_x+tD_x-v_x^{\ 1}) + N_y(O_y+tD_y-v_y^{\ 1}) + N_z(O_z+tD_z-v_z^{\ 1})=0$

$t = ((N\bullet v^1)-(N\bullet O)) / (N\bullet D)$

Ray/sphere intersection

Ray $R=O+tD$

Sphere $S=\{P \mid P \bullet P=r^2\}$



$$(O+tD) \bullet (O+tD) = r^2$$

$$(O_x+tD_x)^2 + (O_y+tD_y)^2 + (O_z+tD_z)^2 = r^2$$

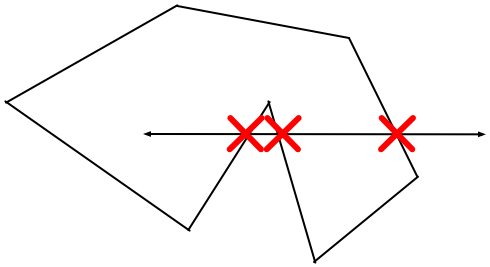
$$(O_x^2+O_y^2+O_z^2) + 2t(O_xD_x+O_yD_y+O_zD_z) + t^2 (D_x^2+D_y^2+D_z^2) - r^2 = 0$$

$$t^2(D \bullet D) + 2t(O \bullet D) + (O \bullet O) - r^2 = 0$$

$$t = \frac{-(O \cdot D) \pm \sqrt{(O \cdot D)^2 - (D \cdot D)((O \cdot O) - r^2)}}{(D \cdot D)}$$

Hit test: Point in nonconvex polygon

Ray casting (1974)



- Odd number of crossings = inside
- Issues:
  - How to find a point that you *know* is inside?
  - What if the ray hits a vertex?
  - Best accelerated by working in 2D
    - You could transform all vertices such that the coordinate system of the polygon has normal = Z axis…
    - Or, you could observe that crossings are invariant under scaling transforms and just project along any axis by ignoring (for example) the Z component.
- Validity proved by the *Jordan curve* theorem

Point in nonconvex polygon

Winding number (1980s)

- The *winding number* of a point P in a curve C is the number of times that the curve wraps around the point.
- For a simple closed curve (as any well-behaved polygon should be) this will be zero if the point is outside the curve, non-zero of it's inside.
- The winding number is the sum of the angles from v^i to P to v^{i+1}.
  - Caveat: This method is elegant but slow.

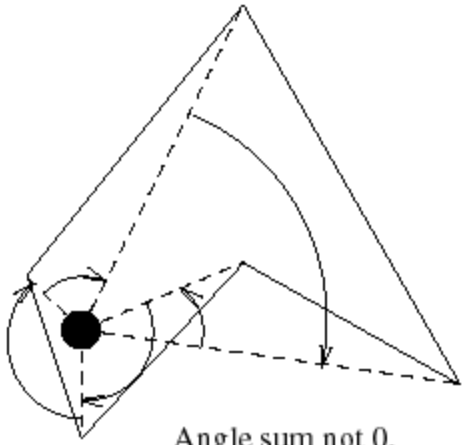
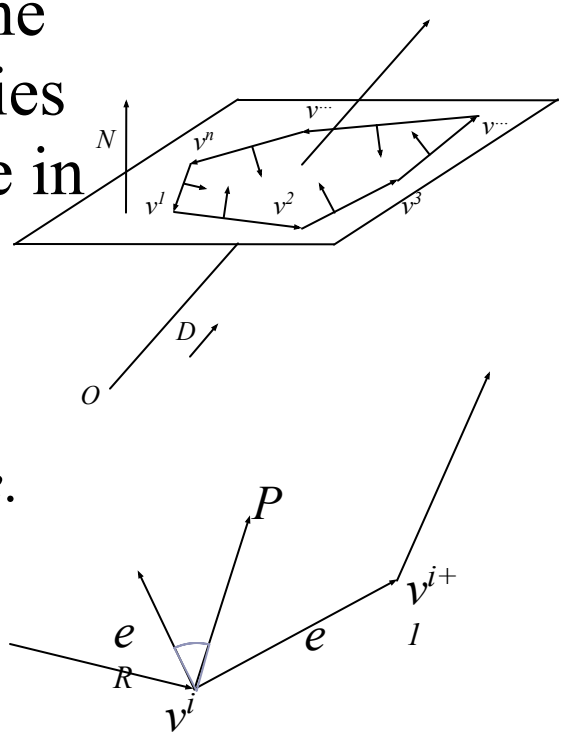


Figure from Eric Haines' "Point in Polygon Strategies", *Graphics Gems IV*, 1994

Point in convex polygon

Half-planes method

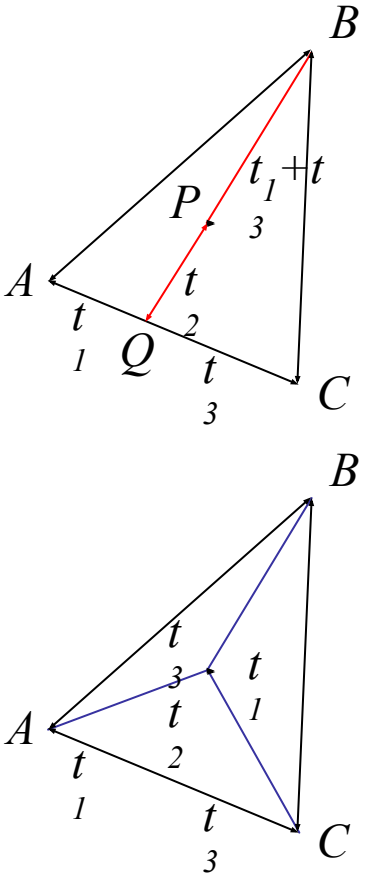
- Each edge defines an infinite half-plane covering the polygon. If the point P lies in all of the half-planes then it must be in the polygon.
- For each edge $e=v^i \rightarrow v^{i+1}$:
  - Rotate e by 90° CCW around N.
  - If $e^R \bullet (P-v^i) < 0$ then the point is outside e.
- Fastest known method.



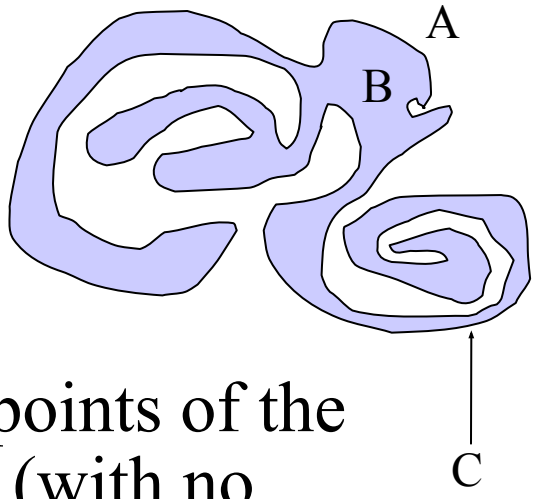
Barycentric coordinates

Barycentric coordinates (t_1,t_2,t_3) are a coordinate system for describing the location of a point P inside a triangle (A, B,C).

- (t_1,t_2,t_3) are the 'masses' to be placed at (A,B,C) respectively so that the center of gravity of the triangle lies at P.
- Interestingly, (t_1,t_2,t_3) are also proportional to the subtriangle areas.



The *Jordan curve theorem*



"Any simple closed curve C divides the points of the plane not on C into two distinct domains (with no points in common) of which C is the common boundary."

- First stated (but proved incorrectly) by Camille Jordan (1838 -1922) in his *Cours d'Analyse*.

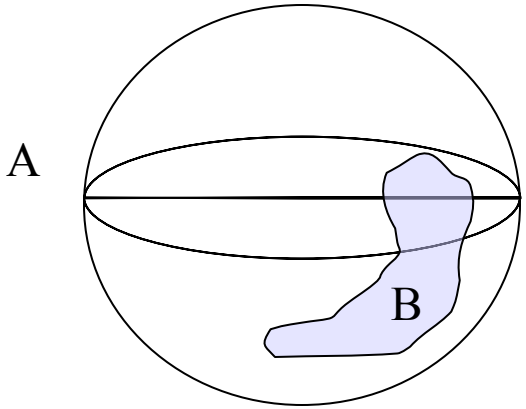
Sketch of proof : (For full proof see Courant & Robbins, 1941.)

- Show that any point in A can be joined to any other point in A by a path which does not cross C, and likewise for B.
- Show that any path connecting a point in A to a point in B *must* cross C.

The Jordan curve theorem on a sphere

Note that the Jordan curve theorem can be extended to a curve on a sphere, or anything which is topologically equivalent to a sphere.

"Any simple closed curve on a sphere separates the surface of the sphere into two distinct regions."



Primitives and world transforms

Given a primitive P and its transform S, is it more efficient to find the intersection in screen space, world space or object space?

- Not screen space: the transform from camera to screen co-ordinates is not affine and therefore not angle-preserving. This would prevent many nice optimizations, such as fast bounding box tests.
- Our maths aren't optimized for world space; it would be nice to have each primitive encoded as statically as possible with minimal parametrization.
  - You only ever need one cube.

Finding the object/ray intersection in object space

Find $R = O+tD$ in object coords:

- S is the local-to-world transform of P.
- Invert S to find S^{-1}, the world-to-local transform.
- Define $O_L=S^{-1}(O)$ and $D_L=S^{-1}(D)$.
- The local ray: $R_L = O_L + t'D_L$
- Solve for t' and find the world hit point at $S(R_L(t'))$.

Wyvill (1995) (Part 2, p.45) compared the floating-point ops required to hit a sphere with a ray in world or local coordinates. He found that at the time it was actually 37% more efficient, per ray, to intersect in local space.

Transforming the normal to the surface

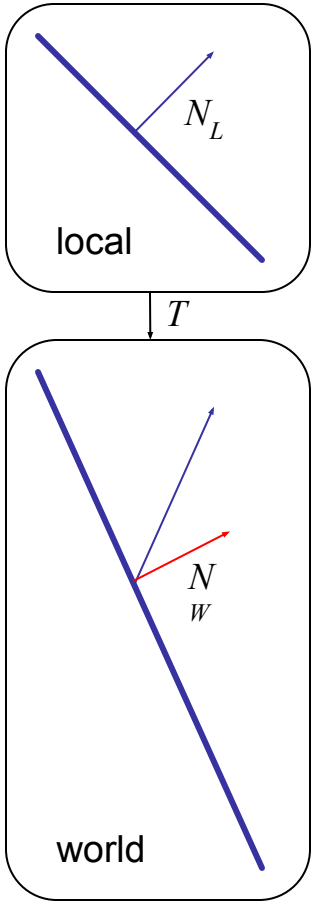
Can we transform the normal by S?

- If *S* is just a concatenated sequence of rotates and translates then the normal can be transformed by *S* as above.
- Scales make things trickier.

To find the world-space normal, multiply the local normal by the *transpose of the inverse* of *S:*

$$N=(S^{-1})^T N_L$$

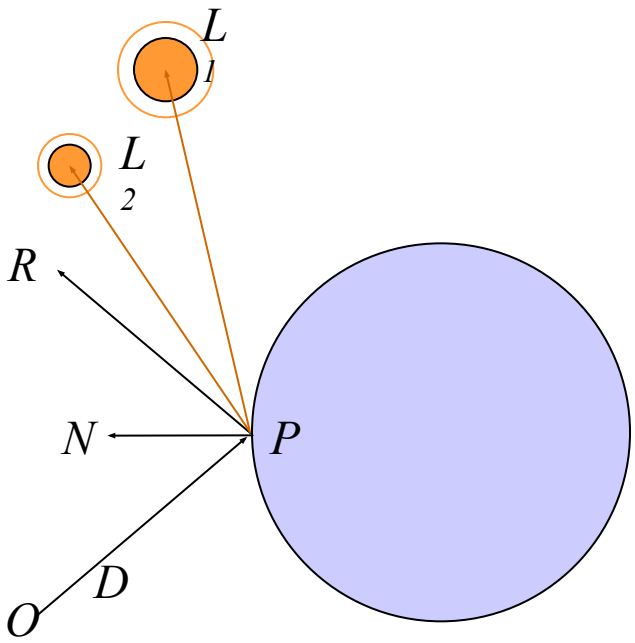
- Can ignore translations
- For any rotation Q, $(Q^{-1})^T=Q$
- Scaling is unaffected by transpose, and a scale of (a,b,c) becomes $(1/a,1/b,1/c)$ when inverted



Lighting revisited

We approximate lighting as the sum of the *ambient, diffuse,* and *specular* components of the light reflected to the eye.

- Associate scalar parameters k_A, k_D and k_S with the surface.
- Calculate diffuse and specular from each light source separately.



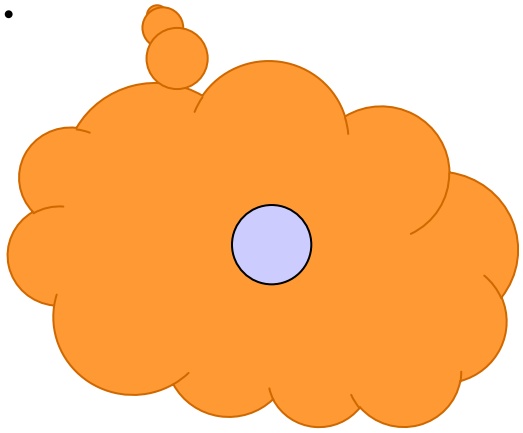
Ambient lighting

Ambient light is a flat scalar constant, L_A.

- The amount of ambient light L_A is a parameter of the scene; the way it illuminates a particular surface is a parameter of the surface.
- Some surfaces (ex: cotton wool) have high ambient coefficient k_A; others (ex: steel tabletop) have low k_A.

Lighting intensity for ambient light alone:

$$I_A(P) = k_A L_A$$



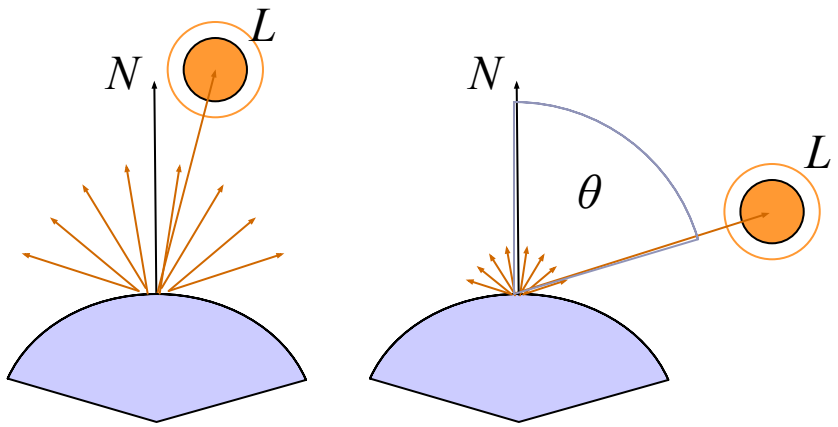
Diffuse lighting

The *diffuse* coefficient k_D measures how much light *scatters* off the surface.

- Some surfaces (e.g. skin) have high k_D, scattering light from many microscopic facets and breaks.
- Others (e.g. ball bearings) have low k_D.

Diffuse lighting intensity:

$$I_D(P) = k_D L_D(cos\ \theta)$$
$$= k_D L_D(N \bullet L)$$



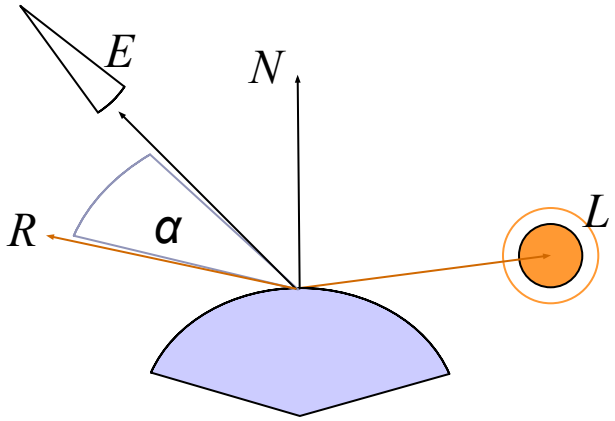
Specular lighting

The *specular* coefficient k_S measures how much light *reflects* off the surface.

- A ball bearing has high k_S; I don't.
- 'Shininess' is approximated by a scalar power n.

Specular lighting intensity:

$$I_S(P) = k_S L_S(\cos\alpha)^n$$
$$= k_S L_S(R \bullet E)^n$$
$$= k_S L_S(2(L \bullet N)N \times L) \bullet E)^n$$

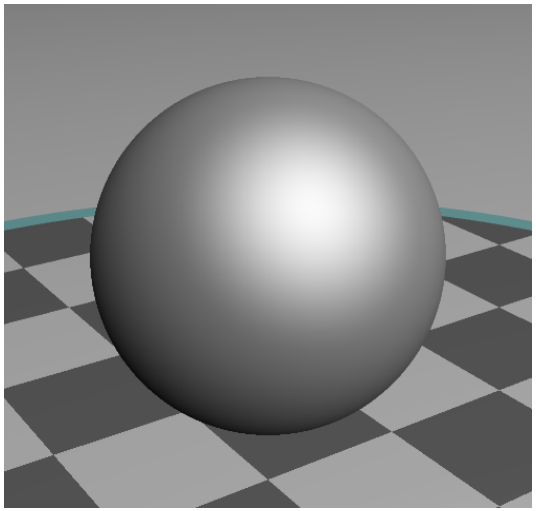


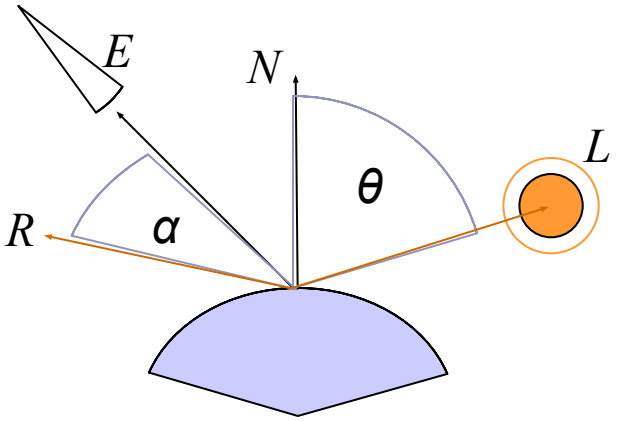
Total illumination

The *total illumination at P* is therefore:

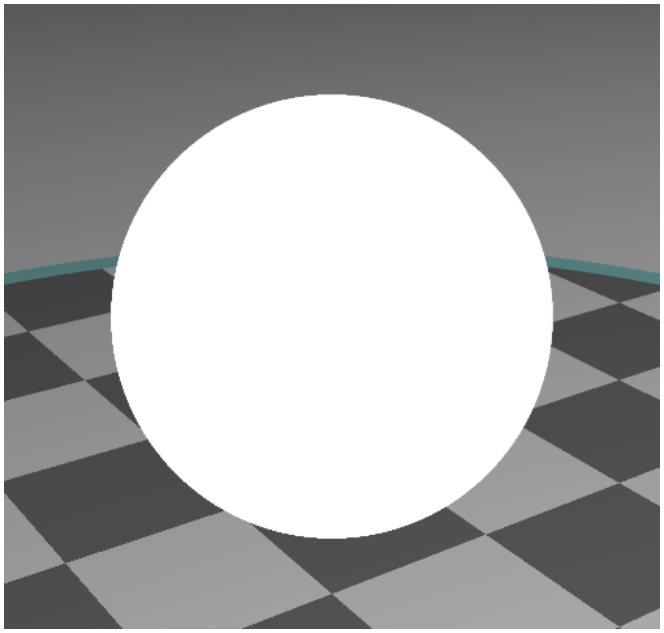
$$I(P) = k_A L_A + k_D L_D (N \bullet L) + k_S L_S (R \bullet E)^n$$

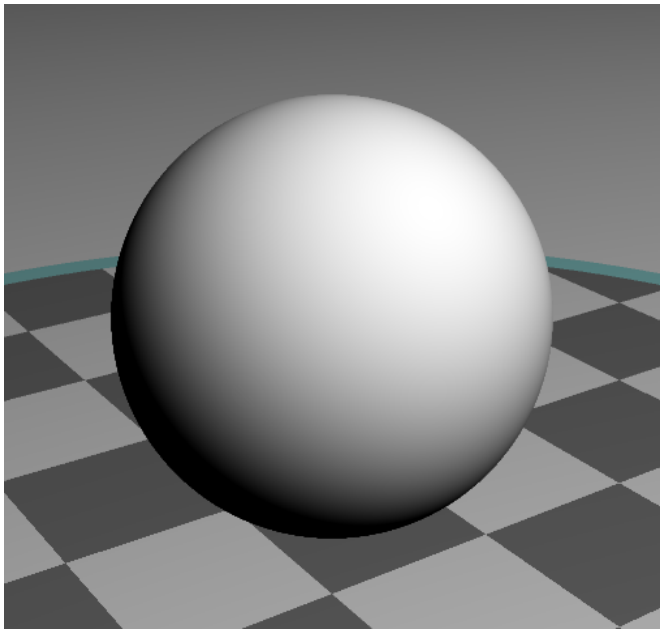
summed over all lights L





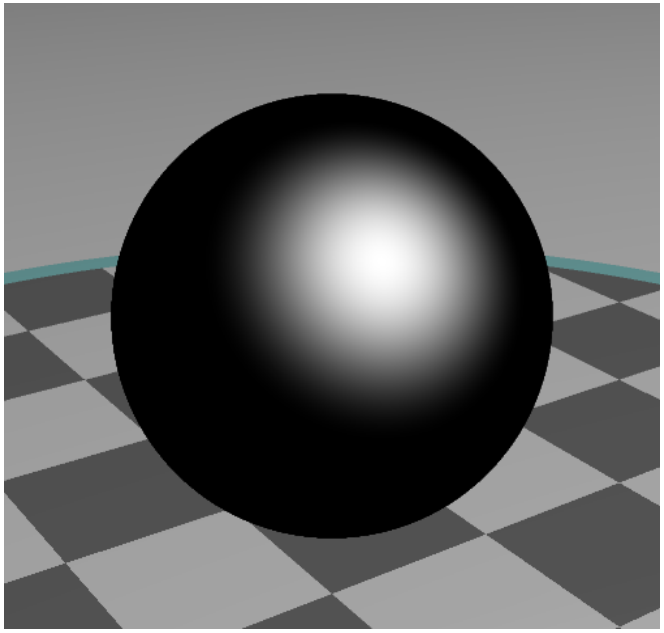
Ambient=1
Diffuse=0
Specular=0

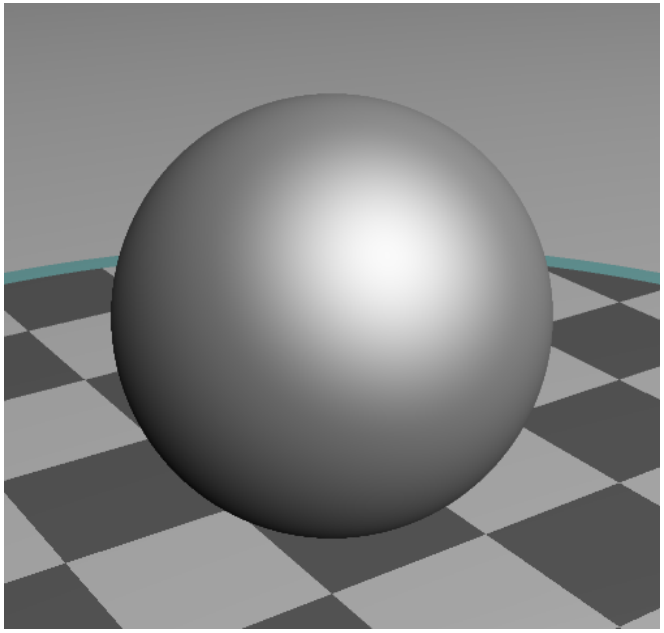




Ambient=0
Diffuse=1
Specular=0

Ambient=0
Diffuse=0
Specular=1
(^{n}=2)



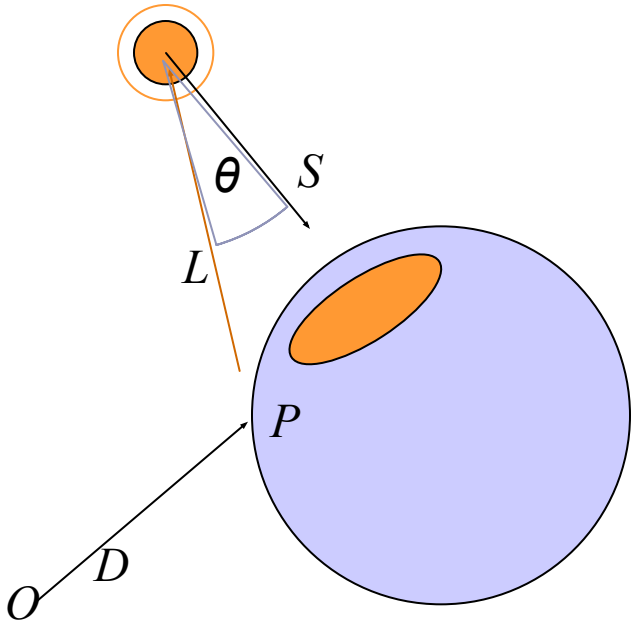


Ambient=0.2
Diffuse=0.4
Specular=0.4
(^{n}=2)

Spotlights

To create a spotlight shining along axis *S*, you can multiply the (diffuse+specular) term by $(\max(L \bullet S, 0))^m$.

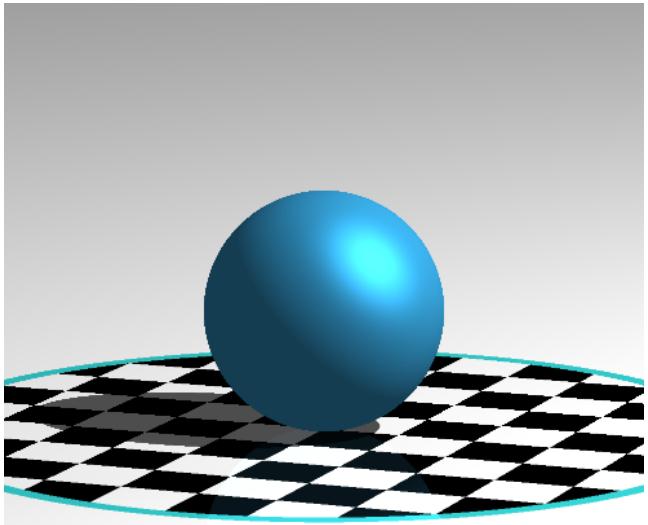
- Raising *m* will tighten the spotlight, but leave the edges soft.
- If you'd prefer a hard-edged spotlight of uniform internal intensity, you can use a conditional, e.g. $((L \bullet S > \cos(15^\circ)) \; ? \; 1 : 0)$.



Shadows

To simulate shadow in ray tracing, fire a ray from P towards each light L_i. If the ray hits another object before the light, then discard L_i in the sum.

- This is a boolean removal, so it will give *hard-edged* shadows.
- Hard-edged shadows imply a pinpoint light source.



Softer shadows

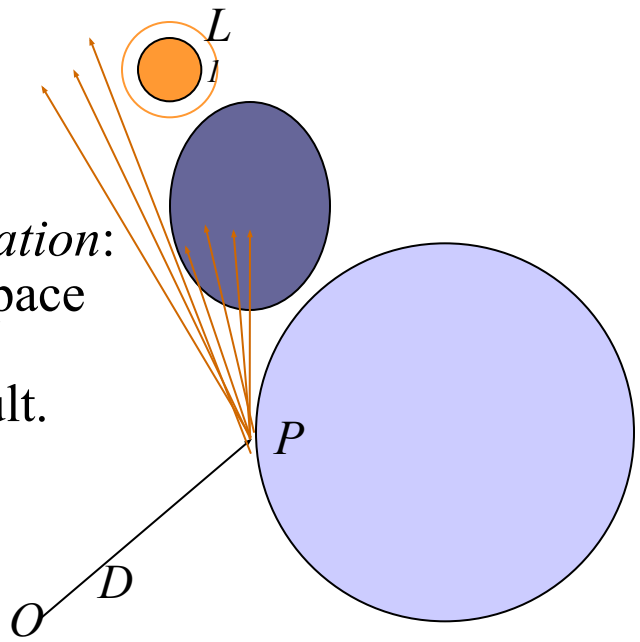
Shadows in nature are not sharp because light sources are not infinitely small.

- Also because light scatters, etc.

For lights with volume, fire many rays, covering the cross-section of your illuminated space.

Illumination is (the total number of rays that aren't blocked) divided by (the total number of rays fired).

- This is an example of *Monte-Carlo integration*: a coarse simulation of an integral over a space by randomly sampling it with many rays.
- The more rays fired, the smoother the result.

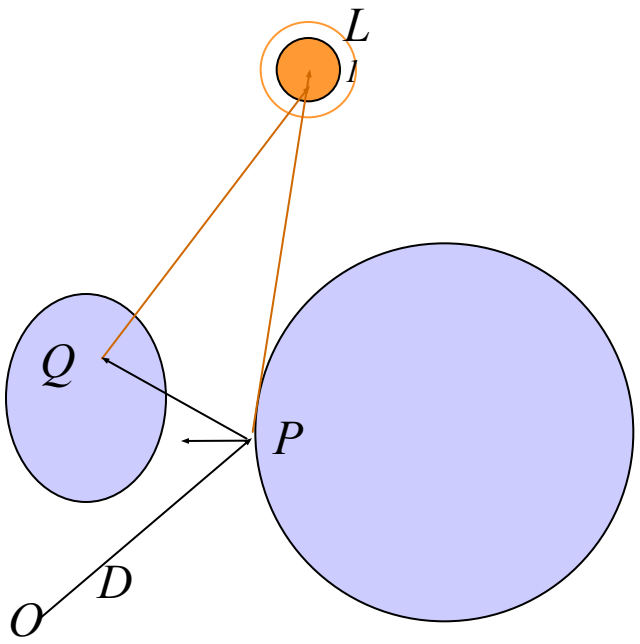


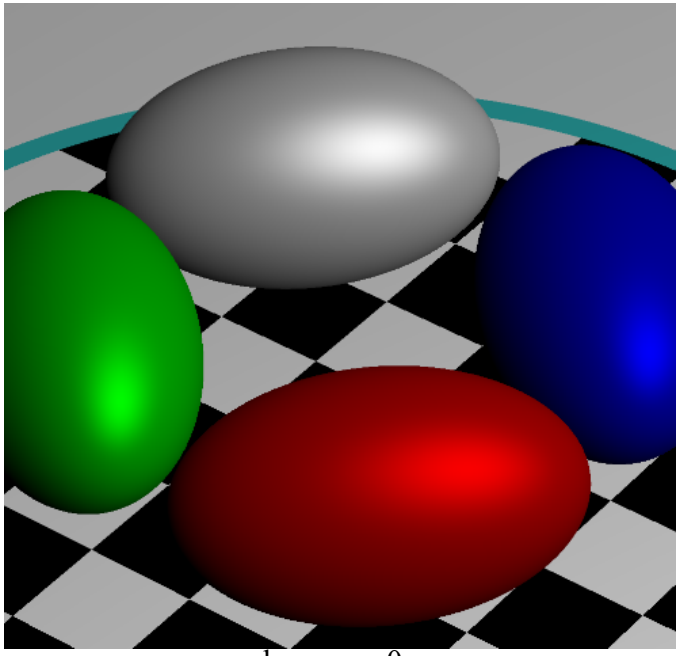
Reflection

Reflection rays are calculated as

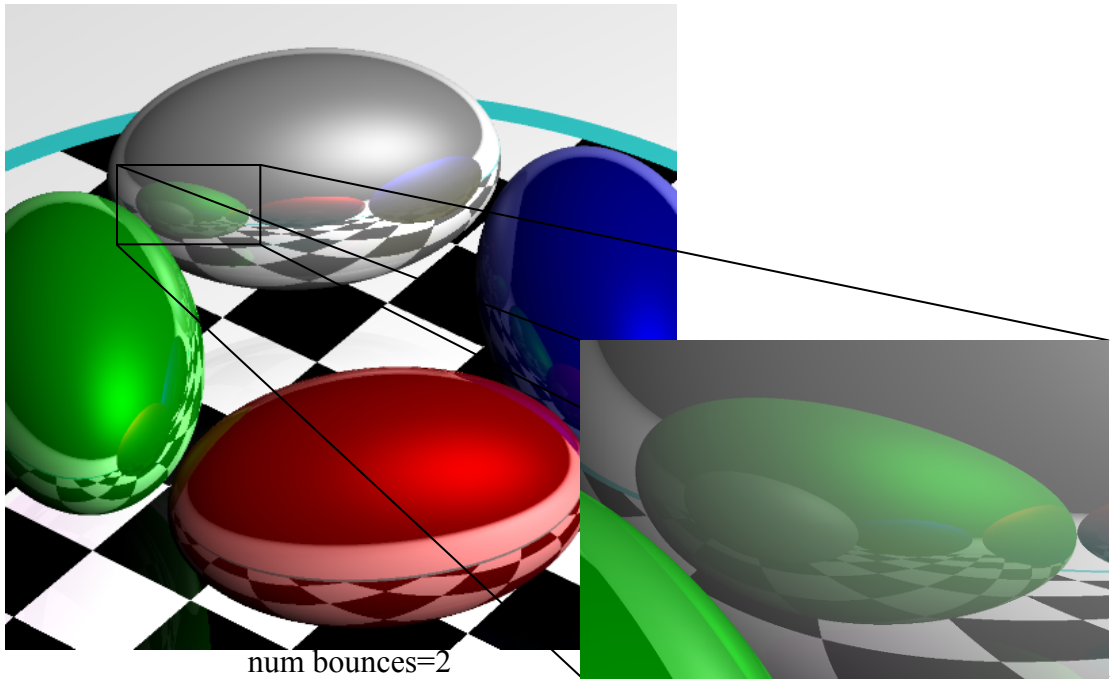
$$R = 2(-D \bullet N)N + D$$

- Finding the reflected color is a recursive raycast.
- Reflection has *scene-dependant* performance impact.

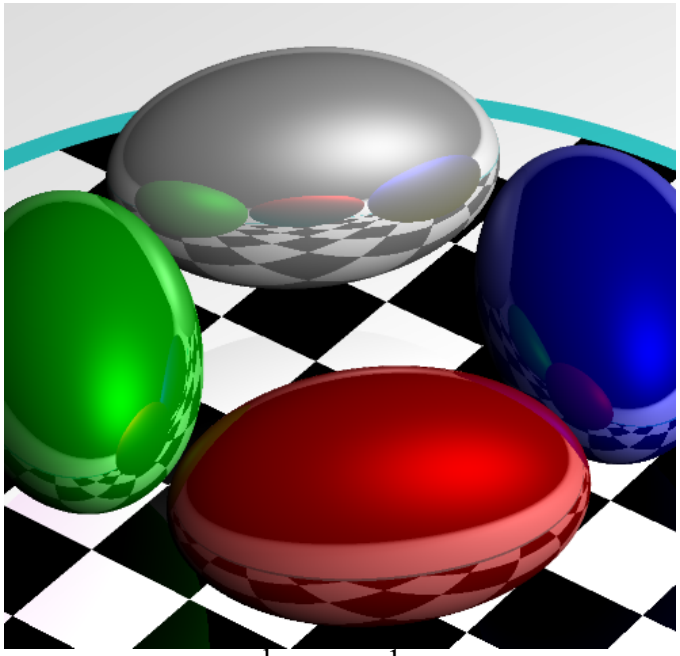




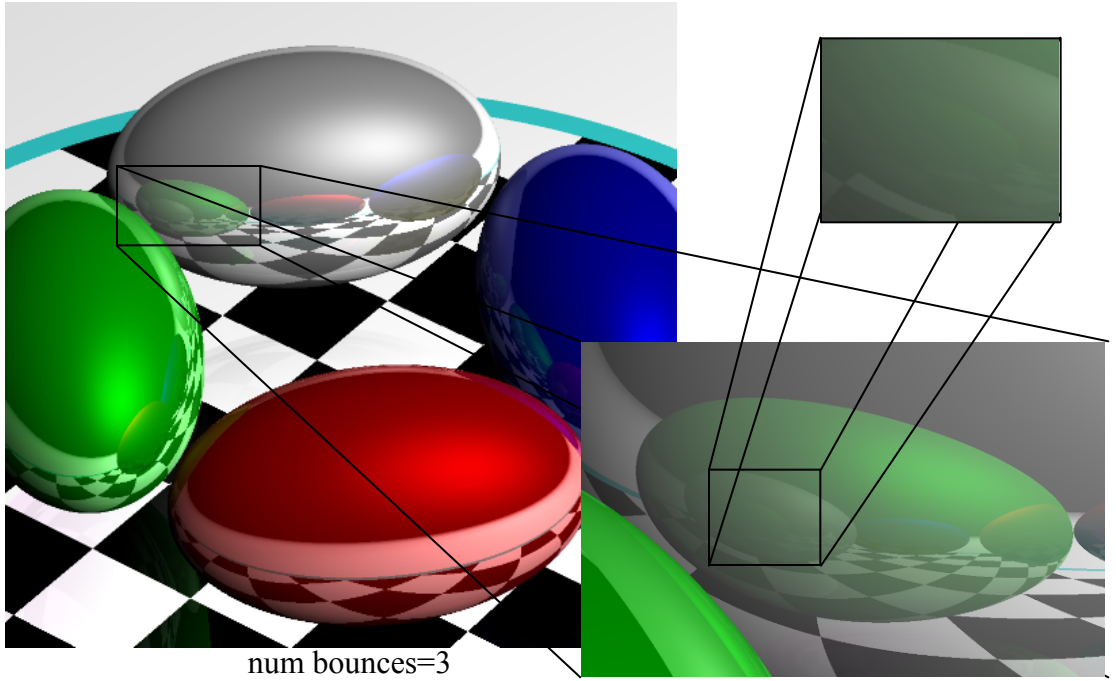
num bounces=0



num bounces=2



num bounces=1

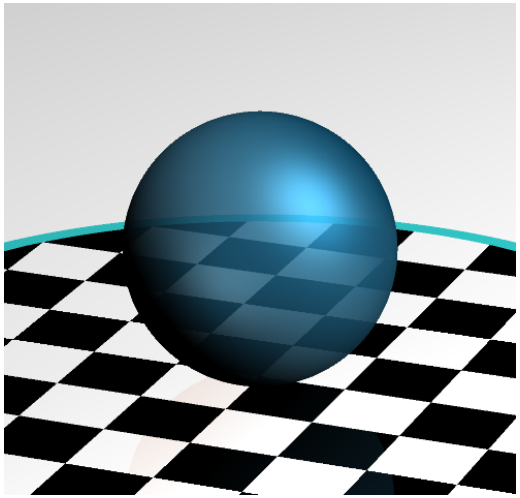


num bounces=3

Transparency

To add transparency, generate and trace a new *transparency ray* with $O_T=P$, $D_T=D$.

To support this in software, make color a 1x4 vector where the fourth component, ‘alpha’, determines the weight of the recursed transparency ray.



Refraction

Snell's Law: $$\frac{sin\,\theta_1}{sin\,\theta_2} = \frac{n_2}{n_1} = \frac{v_1}{v_2}$$

"The ratio of the sines of the *angles of incidence* of a ray of light at the interface between two materials is equal to the inverse ratio of the *refractive indices* of the materials is equal to the ratio of the speeds of light in the materials."

Historical note: this formula has been attributed to Willebrord Snell (1591-1626) and Rene' Descartes (1596-1650) but first discovery goes to Ibn Sahl (940-1000) of Baghdad.

Refraction

The *angle of incidence* of a ray of light where it strikes a surface is the acute angle between the ray and the surface normal.

The *refractive index* of a material is a measure of how much the speed of light[1] is reduced inside the material.

- The refractive index of air is about 1.003.
- The refractive index of water is about 1.33.

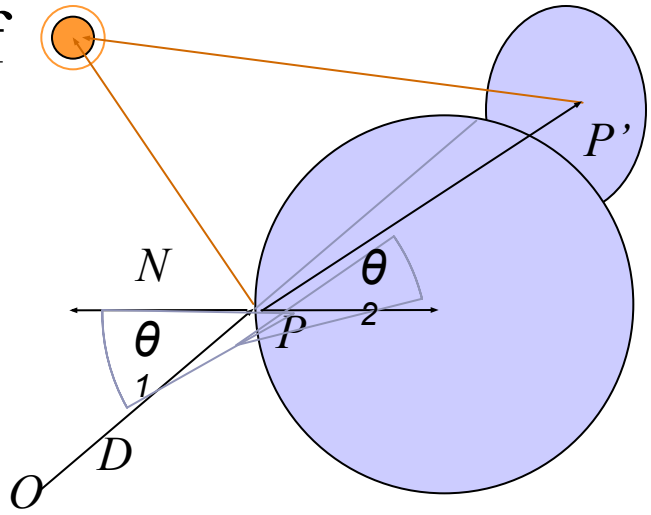
[1] Or sound waves or other waves

Refraction in ray tracing

$$\theta_1 = cos^{-1}(N \bullet D)$$

$$\frac{sin\,\theta_1}{sin\,\theta_2} = \frac{n_2}{n_1} \rightarrow \theta_2 = sin^{-1}\left(\frac{n_1}{n_2} sin\,\theta_1\right)$$

Using Snell's Law and the angle of incidence of the incoming ray, we can calculate the angle from the negative normal to the outbound ray.



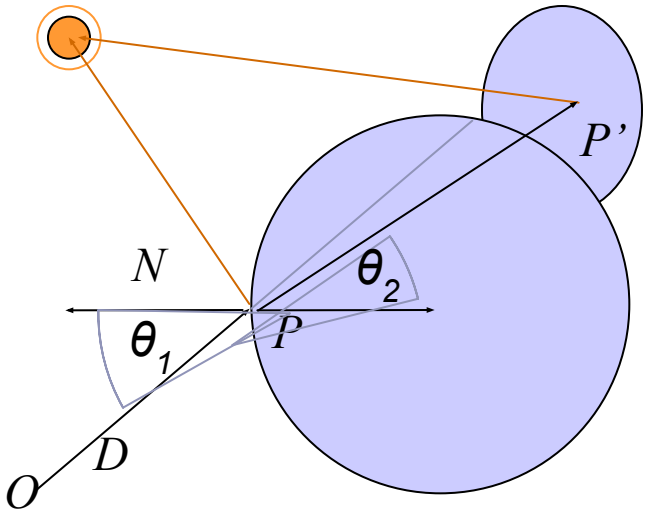
Refraction in ray tracing

What if the arcsin parameter is > 1?

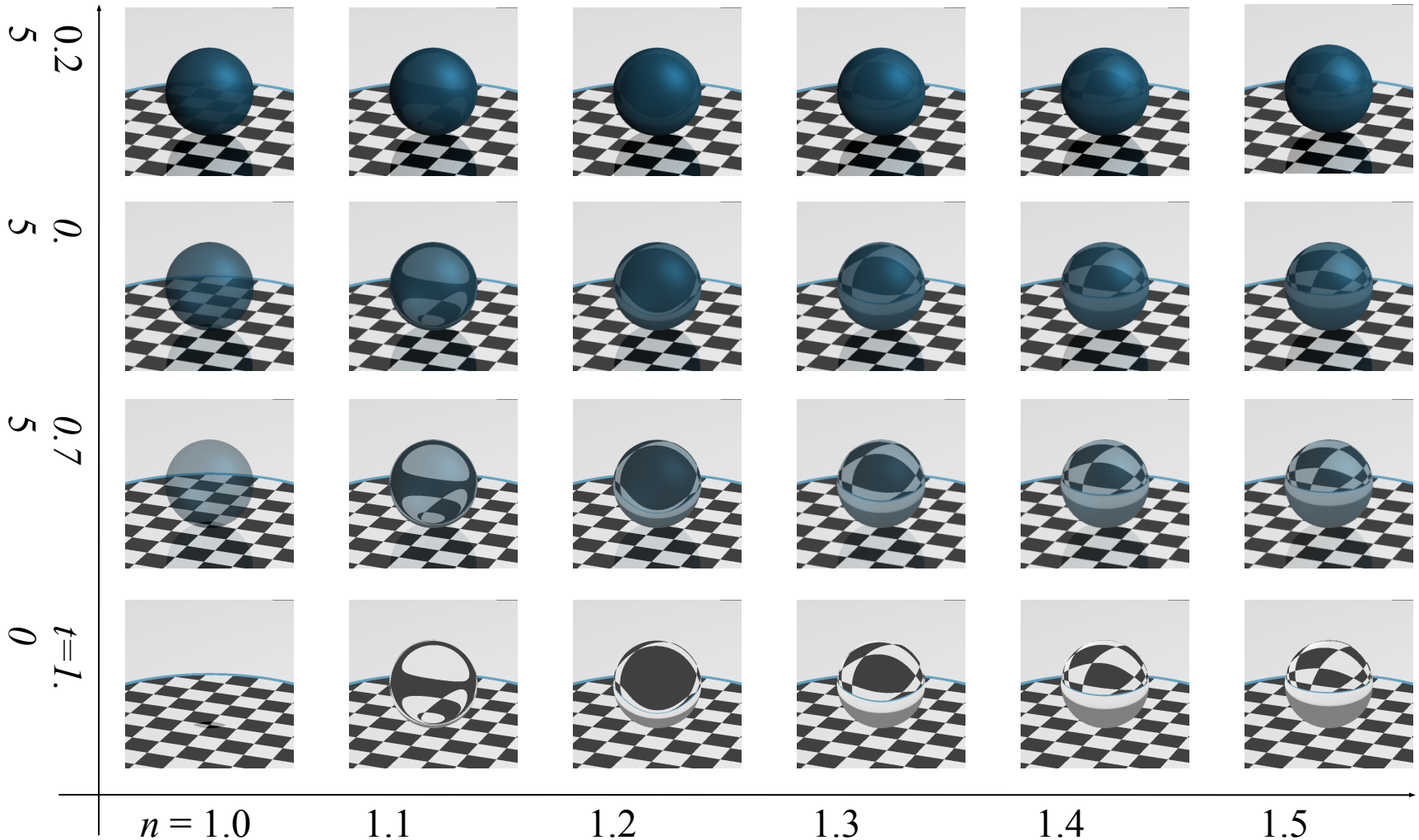
- Remember, arcsin is defined in [-1,1].
- We call this the *angle of total internal reflection*: light is trapped completely inside the surface.

$$\theta_2 = sin^{-1}\left(\frac{n_1}{n_2} sin\,\theta_1\right)$$

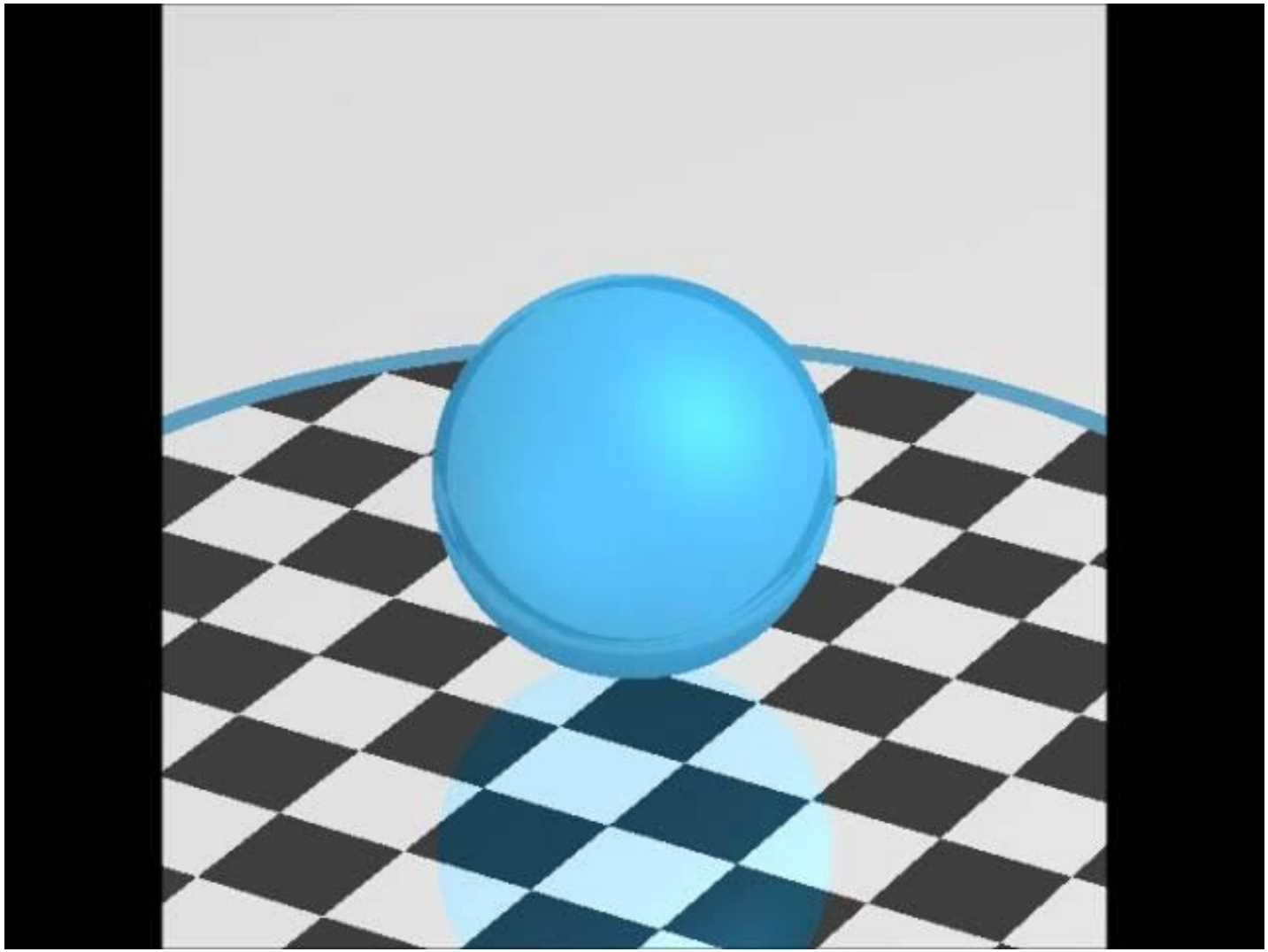
Total internal reflection



Refractive index vs transparency



Refraction in action



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